

Analysis of the Portfolio Constructed by 10 Selected Stocks in Markowitz Model and Index Model with Constraints

Ruixuan Dong*

Ruixuan Dong is with University of California, Irvine, CA 92697 USA

*Corresponding author: ruixuand@uci.edu

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Abstract: Various portfolios appear on the market. People will choose different portfolios as they have varied risk-taking capabilities for portfolios. The purpose of the study is to find the greatest return of portfolios under five constraints by utilizing Markowitz model and Index model. In this article, 10 companies' stocks are selected to form a portfolio. Portfolio with the least risk and portfolio with the greatest return under Markowitz model and Index model are presented respectively. The results of the study show that no matter in Markowitz model or in Index model, the minimum variance portfolios under the constraint that the weight of S&P 500 index is zero has the best performance. Thus, when the weight of S&P 500 Index is zero, the greatest return can be obtained with the least risk-taking both in Markowitz model and Index model. Besides, maximum Sharpe ratio portfolios in Markowitz model and Index model also show the best performance when the weight of S&P 500 Index is zero, which means that the return of portfolio with certain risk-taking is the highest under the constraint that weight of S&P 500 Index is zero. This study could provide a guideline for people when choose their own portfolio.

1. Introduction

With the prevalence of the concept of investment and financial management, an increasing number of people are setting out to make use of investment to obtain high returns. In order to decrease losses of investment and increase investments' returns, people may apply relevant financial knowledge and models to help themselves to investment rationally. Some studies prefer to employ Markowitz model and Index model to reach a conclusion. I Komang Agus Adi Swara Putra and I Made Dana [1] give that optimal portfolio model performance in Index model performs better compared with that in Markowitz model while statistics shows that the average return of portfolio in index model has no obvious difference with average return of portfolio in Markowitz model. Moreover, Tanuj Nandan and Nivedita Srivastava [2] utilizes Markowitz model and Index model to construct an optimal portfolio in Indian market. The goal of this research in the article written by David Moreno et al. [3] is to analyze some risk forecasting models' performance when acquiring the optimal portfolios.

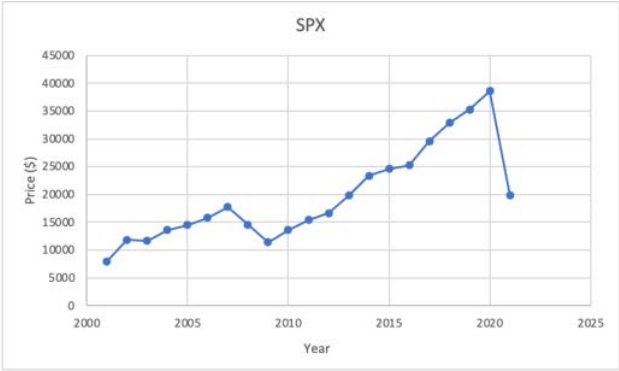
After analyzing the performance of models, the study claims that none of these models can reach a obviously better average performance. However, when it comes to the worst-case performance, the results presents that model s using LPM which is with a high degree of risk aversion perform better than models employing a symmetric homoscedastic or heteroscedastic. Financial models play a significant role in selecting optimal portfolio to obtain the highest return or reduce the risk taking. Josmy Varghese and Anoop Joseph [4] mainly make a comparison between Markowitz model and Sharpe's model and its result contents that Markowitz model relies on the continued probabilistic growth and expansion and Sharpe's single model is helpful to select the optimal portfolio through analysis and calculate the respective weights based on some essential variables. On the contrary, the limitation of Sharpe's single model is that the model ignores the uncertainty in the market over time. The study also concludes some similarities and differences of Markowitz model and Index model. Ritika Sinha and Prabhat Kumar Tripathi [5] analyze and compare Markowitz model and Index model based on some important parameters to help inventors to have a better understanding of

construct Sharpe's optimal model and Markowitz's mean variance portfolio set in Indian market. In addition, the outbreak of COVID-19 has had a certain impact on the economic market. The following articles explore the influence of COVID-19 on the performance of portfolios. Immas Nurhayati et al. [6] provide the result that COVID-19 has a negative impact on the stock market, which results in investors' losses on the portfolios. Therefore, the implication of the study is that investor should regard the outbreak of COVID-19 as a systematic risk factor to evaluate their portfolios' performance and they may need to reconstruct their portfolio. The similar result is also given by Henny Rahyuda [7], which emphasizes that an obvious difference appears between quadrimester I 2019 and 2020 on LQ45 stocks of optimal portfolio in the treynor model, showing the negative impact of COVID-19 on the stock market. Studies that only employ one kind of model demonstrate significant findings as well. Mokta Rani Sarker [8] emphasizes how to construct the optimal portfolio from the stocks of 164 companies by using Markowitz model. The study of Alpesh Gajera and Kaushal Thkar [9] mainly focuses on creating an optimal portfolio in Index model by using the data of real market date of SENSEX 30 Securities. Models are always employed to evaluate the performance of portfolio. Chintan A. Shah [10] makes use of Sharpe model to compare the performance of BSE15 securities. By analyzing the data offered by Sharpe model, the researcher claims that the loss can be covered from other securities which are included in the portfolio.

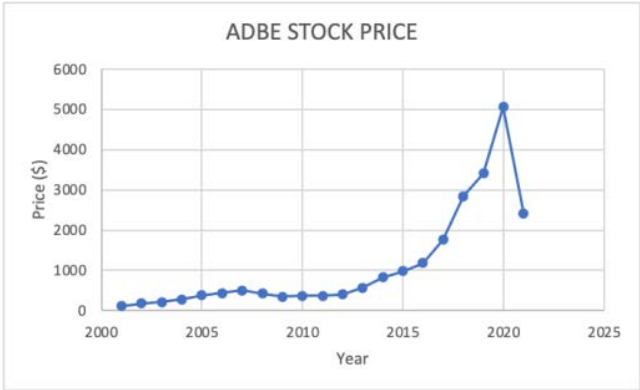
The article mainly demonstrates the minimum variance portfolio and maximum Sharpe ratio portfolio in Markowitz model and Index model respectively. The portfolio contains 10 selected companies' stocks. The greatest returns of portfolios under different constraints in Markowitz model and Index model are provided by the statistics shown on the table. Statistics gives the result that the greatest return of portfolio in Markowitz model and Index model are under same constraint. More detailed, the findings demonstrate that the highest return of minimum variance portfolio and maximum Sharpe ratio portfolio can be obtained no matter in Markowitz model or Index model when the weight of S&P 500 index is zero under the premise of least risk-taking while the lowest returns of portfolio are under different constraint according to different models.

2. Method

In this article, the portfolio includes the stocks of 10 selected companies: Adobe Inc. (ADBE), International Business Machines Corporation (IBM), SAP SE (SAP), Bank of America Corporation (BAC), Citigroup Inc.(C), Wells Fargo & Company (WFC), Travelers Companies, Inc (TRV), Southwest Airlines Co. (LUV), Alaska Air Group (ALK) and Hawaiian Airlines (HA). Besides, S&P 500 index is also utilized in the study. The 10 companies are mainly selected from three industries, which are computer software, finance, and airline. Computer software industry contains the following three companies. Adobe Inc. (ADBE) is an American multinational computer software company. International Business Machines Corporation (IBM) is an American multinational technology corporation, and SAP SE (SAP) is a German multinational software corporation. The four selected companies in finance industry are as follows. Bank of America Corporation (BAC) is an American multinational investment bank and financial services holding company. Citigroup Inc. (C) is an American multinational investment bank and financial services corporation and Wells Fargo & Company (WFC) is an American multinational financial services company. Travelers Companies Inc. (TRV) is an American insurance company. There are also some selected companies in the airline industry. Southwest Airlines Co. (LUV) is one of the major airline of the United States. Alaska Air Group (ALK) is an airline holding company and Hawaiian Airline (HA) is one of the largest commercial airline. Statistics of stock price employed in this study from May 11st, 2001 to May 12nd, 2021. Figure 1 shows the trend of companies stock prices.



(a)



(b)



(c)



(d)



(e)



(f)



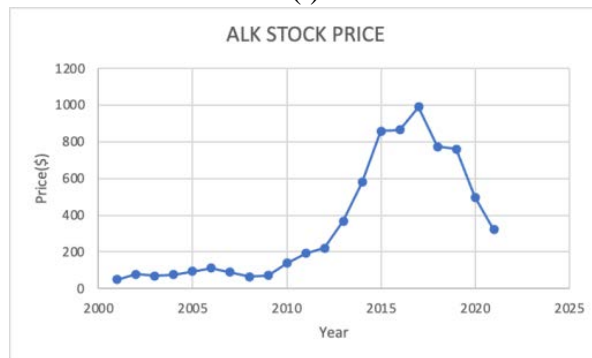
(g)



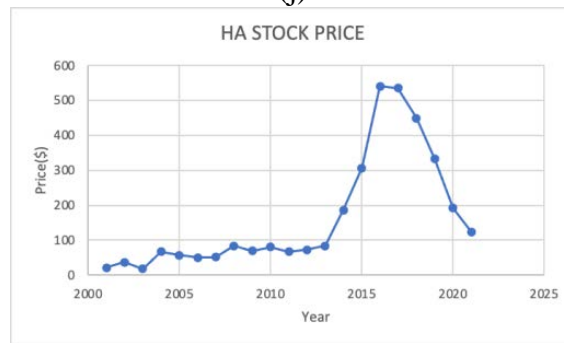
(h)



(i)



(j)



(k)

Fig. 1. Stock price for individual stocks and SPX.(a)stock price for S&P 500. (b)stock price for Adobe Inc.(c)stock price for International Business Machines Corporation.(d)stock price for SAP SE.(e)stock price for Bank of America Corporation.(f)stock price for Citigroup Inc.(g)stock price for Wells Fargo & Company.(h)stock price for Travelers Companies Inc.(i)stock price for Southwest Airlines Co.(j)stock price for Alaska Air Group.(k)stock price for Hawaiian Airline.

Source: Yahoo Finance.

Table 1 is the correlation between the selected stocks. The statistics shown on the table 2 emphasizes that the correlation between the selected stocks are relatively low, indicating that the portfolio analyzed in the paper has lower risk, and associates with a better diversification effect. The low correlation shows that stocks do not perform in the same way and they do not interfere with each other. Besides, the low correlation in the portfolio decreases the volatility of the whole portfolio and make the portfolio more optimal.The low correlation is effective in decreasing the risk -taking and maintaining a stable return to certain extent.

Table 1. Correlation between Stock Return

COR	SPX	ADBE	IBM	SAP	BAC	C	WFC	TRV	LUV	ALK	HA
SPX	1.00	0.66	0.65	0.65	0.60	0.70	0.55	0.60	0.53	0.46	0.39
ADBE	0.66	1.00	0.45	0.53	0.42	0.46	0.30	0.45	0.39	0.23	0.18
IBM	0.65	0.45	1.00	0.58	0.31	0.42	0.27	0.38	0.35	0.35	0.25
SAP	0.65	0.53	0.58	1.00	0.33	0.43	0.30	0.37	0.32	0.28	0.14
BAC	0.60	0.42	0.31	0.33	1.00	0.83	0.76	0.39	0.43	0.27	0.34
C	0.70	0.45	0.42	0.43	0.83	1.00	0.70	0.51	0.42	0.30	0.34
WFC	0.55	0.30	0.27	0.30	0.76	0.70	1.00	0.34	0.40	0.35	0.36
TRV	0.60	0.45	0.38	0.38	0.40	0.51	0.34	1.00	0.40	0.36	0.24
LUV	0.53	0.39	0.35	0.32	0.43	0.43	0.40	0.40	1.00	0.52	0.42
ALK	0.46	0.23	0.35	0.28	0.28	0.30	0.35	0.36	0.52	1.00	0.40
HA	0.39	0.18	0.25	0.14	0.34	0.34	0.36	0.24	0.42	0.40	1.00

Note: COR stands for correlation.

In this study, five constraints are utilized to find whether the greatest returns in Markowitz model and Index model are subject to different constraints or the highest returns in these two model are under same constraint. Below are the formula of the constraints.

Constraint 1: No Constraint

Constraint 2: $\sum_{i=1}^7 |w_i| \leq 2$

Constraint 3: $|w_i| \leq 1$, for \forall_i

Constraint 4: $w_i \geq 0$, for \forall_i

Constraint 5: $w_i = 0$

The detailed explanation of constraints are as follows. The content of constraint 1 is no constraint. The formula of constraint 2 means that the absolute value of sum of stocks' weights is no more than 2. Constraint 3 provides the limit that absolute value of each stock's weight is no more than 1. The content of constraint 4 is that stock's weight is more than or equal to zero for each stock. Constraint 5 which is the last one shows that weight of S&P 500 Index equals 0.

Besides, Markowitz model Index model are employed in the study. These two models play significant roles in obtaining the greatest return of portfolio. Markowitz model, one of the model utilized in the analysis, is also called mean-variance model. Markowitz model is a portfolio optimization model, which assists in the selection of the most efficient portfolio by analyzing various possible portfolios of the given securities. The other model is called Index model. The single-index model is a simple asset pricing model to measure both the risk and the return of a stock. The equation of index model show that the stock return is influenced by the market (beta), has a firm specific expected value (alpha) and firm-specific unexpected component (residual). In this study, Markowitz model and Index model are main two model helps to analyze the performance of the portfolio and to obtain the highest return of the portfolio. Additionally, the essential formula applied in the study is offered below.

Portfolio = $W(\text{SPX}) * \text{SPX} + W(\text{ADBE}) * \text{ADBE} + W(\text{IBM}) * \text{IBM} + W(\text{SAP}) * \text{SAP} + W(\text{BAC}) * \text{BAC} + W(\text{C}) * \text{C} + W(\text{WFC}) * \text{WFC} + W(\text{TRV}) * \text{TRV} + W(\text{LUV}) * \text{LUV} + W(\text{ALK}) * \text{ALK} + W(\text{HA}) * \text{HA}$.

In the formula, $W(\text{SPX})$ stands for the weight of S&P 500 Index and SPX stands for the stock price of S&P 500 index. $W(\text{ADBE})$ stands for the weight of Adobe Inc. and ADBE stands for the stock price of Adobe Inc. $W(\text{IBM})$ stands for the weight of and IBM stands for the stock price of International Business Machines Corporation. $W(\text{SAP})$ stands for the weight of SAP SE and SAP stands for the stock price of SAP SE. $W(\text{BAC})$ stands for the weight of Bank of America Corporation and BAC stands for the stock price of Bank of America Corporation. $W(\text{C})$ stands for the weight of Citigroup Inc and C stands for the stock price of Citigroup Inc. $W(\text{WFC})$ stands for the weight of Wells Fargo & Company and WFC stands for the stock price of Wells Fargo & Company. $W(\text{TRV})$ stands for the weight of and WFC stands for the stock price of. $W(\text{TRV})$ stands for the weight of Travelers Companies Inc and WFC stands for the stock price of Travelers Company Inc. $W(\text{LUV})$ stands for the weight of Southwest Airlines Co and LUV stands for the stock price of Southwest Airlines Co. $W(\text{ALK})$

stands for the weight of Alaska Air Group and ALK stands for the stock price of Alaska Air Group. W(HA) stands for the weight of Hawaiian Airlines and HA stands for the stock price of Hawaiian Airlines.

Furthermore, two kind of portfolios are analyzed in the article. One of the portfolios is minimum variance portfolio. It indicates a well-diversified portfolio that consists of individually risky assets, which are hedged when traded together, resulting in the lowest possible risk for the rate of expected return. The low correlation of minimum variance portfolio allows the portfolio to be less risky. The other one is maximum Sharpe ratio portfolio. In this portfolio, Sharpe ratio is an indicator to measure the risk-return ratio.

3. Result

A. Minimum Variance Portfolio under Markowitz Model and Index Model

Table 2 shows the weights of minimum variance portfolios under five constraints in Markowitz model. Table 3 provides the returns of minimum variance portfolio in Markowitz model. According to table 3, the return ratio of portfolio under constraint 5 is 9.38%, which means that minimum variance portfolio under the constraint that the weight of S&P 500 index is zero in Markowitz model under the premise of the least risk-taking has the greatest return. In the contrary, return of portfolio under constraint 2 has the lowest value, showing that portfolio under the constraint that the absolute value of sum of stocks' weights is no more than 2 has the least return, 6.71%., compared with portfolios under other constraints.

Table 2. Weights of Minimum Variance Portfolio in Markowitz Model

	SPX	ADBE	IBM	SAP	BAC	C	WFC	TRV	LUV	ALK	HA
C1	1.11	-0.10	0.05	-0.10	0.01	-0.23	0.14	0.20	-0.01	-0.05	-0.03
C2	1.11	-0.10	0.05	-0.09	0.01	-0.23	0.14	0.19	-0.01	-0.05	-0.03
C3	1.00	-0.09	0.08	-0.09	0.01	-0.23	0.16	0.23	0.01	-0.05	-0.03
C4	0.84	0.00	0.00	0.00	0.00	0.00	0.00	0.16	0.00	0.00	1.52
C5	0.00	0.04	0.34	-0.02	0.01	0.35	0.35	0.53	0.05	-0.03	-0.02

Note: C1 stands for constraint 1. C2 stands for constraint 2. C3 stands for constraint 3. C4 stands for constraint 4. C5 stands for constraint 5.

Table 3. Key Results of Minimum Variance Portfolio in Markowitz Model

	Return	Standard Deviation	Sharpe Ratio
C1	6.69%	11.75%	56.97%
C2	6.71%	11.75%	57.08%
C3	6.96%	11.79%	59.10%
C4	7.80%	14.61%	53.34%
C5	9.38%	15.45%	60.72%

Note: C1 stands for constraint 1. C2 stands for constraint 2. C3 stands for constraint 3. C4 stands for constraint 4. C5 stands for constraint 5.

Table 4 presents the weights of minimum variance portfolio under five constraints in Index model. Table 5 is the returns of minimum variance portfolio in index model. Based on table 4, the highest return ratio is 9.26%. Therefore, portfolio with the constraint that the weight of S&P 500 index is zero has the greatest return with the least risk-taking, which indicates the portfolio shows the best performance when the weight of S&P 500 index equals zero. Portfolio without constraint has the lowest return ratio which is 5.85%.

Table 4. Weights of Minimum Variance Portfolio in Index Model

	SPX	ADBE	IBM	SAP	BAC	C	WFC	TRV	LUV	ALK	HA
C1	1.44	-0.11	-0.01	--0.10	-0.09	-0.16	-0.01	0.11	-0.03	-0.02	-0.03
C2	1.39	-0.01	0.00	-0.97	-0.08	-0.15	-0.01	0.11	-0.02	-0.02	-0.03
C3	1.00	-0.07	0.09	-0.07	-0.06	-0.14	0.04	0.23	0.01	0.00	-0.02
C4	0.84	0.00	0.00	0.00	0.00	0.00	0.00	0.16	0.00	0.00	0.00
C5	0.00	0.03	0.29	0.01	-0.02	-0.10	0.15	0.50	0.09	0.05	-0.01

Note: C1 stands for constraint 1. C2 stands for constraint 2. C3 stands for constraint 3. C4 stands for constraint 4. C5 stands for constraint 5.

Table 5. Key Results of Minimum Variance Portfolio in Index Model

	Return	Standard Deviation	Sharpe Ratio
C1	5.85%	11.95%	48.97%
C2	6.07%	11.96%	50.76%
C3	6.90%	12.47%	55.32%
C4	7.80%	14.61%	53.34%
C5	9.26%	16.64%	55.62%

Note: C1 stands for constraint 1. C2 stands for constraint 2. C3 stands for constraint 3. C4 stands for constraint 4. C5 stands for constraint 5.

B. Maximum Sharpe Ratio portfolio under Markowitz model and Index model

Weights of maximum Sharpe ratio portfolio in Markowitz model is in table 6 and table 7 presents the returns of maximum Sharpe ratio portfolio in Markowitz model. The greatest return of maximum sharpe ratio in Markowitz model is 26.53%, which is under constraint 5. This means that when the portfolio's risk adjusted performance is the best, the return is 26.53%, the greatest of that the five constraints. Additionally, the least return of maximum Sharpe ratio portfolio in the Markowitz model is 17.60%. Table 8 includes the values of weights of maximum Sharpe ratio in Index model. Table 9 is the returns of maximum Sharpe ratio portfolio in Index model. In the Index model, the greatest return whose value is 23.79% of maximum Sharpe ratio portfolio is under constraint 5 and the least return is owned by portfolio under constraint 4, with the value of 17.28%.

Table 6. Weights of Maximum Sharpe Ratio Portfolio in the Markowitz Model

	SPX	ADBE	IBM	SAP	BAC	C	WFC	TRV	LUV	ALK	HA
C1	0.05	0.36	-0.20	0.07	0.30	0.66	0.21	0.36	-0.15	0.13	0.11
C2	0.40	0.28	-0.01	0.01	0.17	-0.48	0.20	0.30	-0.02	0.08	0.08
C3	0.50	0.36	-0.22	0.07	0.30	-0.66	0.21	0.36	-0.15	0.13	0.11
C4	0.00	0.50	0.00	0.00	0.00	0.00	0.00	0.18	0.00	0.20	0.13
C5	0.00	0.51	-0.20	0.14	0.37	-0.77	0.29	0.50	-0.17	0.18	0.15

Note: C1 stands for constraint 1. C2 stands for constraint 2. C3 stands for constraint 3. C4 stands for constraint 4. C5 stands for constraint 5.

Table 7. Key Results of Maximum Sharpe Ratio Portfolio in Markowitz Model

	Return	Standard Deviation	Sharpe Ratio
C1	22.07%	21.33%	103.45%
C2	17.60%	17.70%	99.40%
C3	22.01%	21.33%	103.45%
C4	18.24%	24.75%	73.68%
C5	26.53%	25.98%	102.11%

Note: C1 stands for constraint 1. C2 stands for constraint 2. C3 stands for constraint 3. C4 stands for constraint 4. C5 stands for constraint 5.

Table 8. Weights of Maximum Sharpe Ratio Portfolio in the Index Model

	SPX	ADBE	IBM	SAP	BAC	C	WFC	TRV	LUV	ALK	HA
C1	0.55	0.38	-0.23	0.30	-0.02	-0.38	0.02	0.29	0.04	0.19	0.11
C2	0.48	0.48	0.36	-0.15	0.02	-0.01	-0.35	0.40	0.28	0.04	0.10
C3	0.55	0.38	-0.23	0.30	-0.02	-0.40	0.04	0.29	0.04	0.19	0.11
C4	0.00	0.41	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.21	0.12
C5	0.00	0.53	-0.21	0.09	0.01	-0.41	0.10	0.42	0.08	0.25	0.14

Note: C1 stands for constraint 1. C2 stands for constraint 2. C3 stands for constraint 3. C4 stands for constraint 4. C5 stands for constraint 5.

Table 9. Key Results of Maximum Sharpe Ratio Portfolio in Index Model

	Return	Standard Deviation	Sharpe Ratio
C1	19.81%	21.99%	90.11%
C2	18.90%	21.04%	89.83%
C3	19.81%	21.99%	90.11%
C4	17.28%	23.35%	73.98%
C5	23.79%	26.68%	89.17%

Note: C1 stands for constraint 1. C2 stands for constraint 2. C3 stands for constraint 3. C4 stands for constraint 4. C5 stands for constraint 5.

4. Conclusion

Among the minimum variance portfolios under five constraints in Markowitz model and Index model, portfolio under constraint 5 has the same and greatest return ratio in these two model. It shows that the highest return of portfolio can be obtained both in Markowitz model and Index model when the weight of S&P 500 index is zero under the premise of least risk-taking. The lowest return ratio is under different constraints. To be more specific, the lowest return ratio of minimum variance portfolio in Markowitz model is under constraint 2 and the lowest return ratio of minimum variance portfolio in Index model is under constraint 1. Therefore, under the premise of least risk-taking, the lowest return is presented in Markowitz model when the sum of stocks' weights less than or equal to 2 and the lowest return is presented in Index model when there is no constraint. When it comes to maximum Sharpe ratio portfolios, same results are shown in the Markowitz model and Index model. Portfolios under constraint 5 has the greatest return both in two models and the least returns of portfolios are both under constraint 4 in two models. To be more detailed, both in Markowitz model and Index model the greatest returns can be obtained when the weight of S&P 500 equals zero and the lowest returns are provided when each stock's weight is more than zero. In a conclusion, portfolio has the greatest return when the weight of S&P 500 index equals zero. The research result may be skewed due to the impact of COVID-19 on the whole economic market.

References

- [1] I. Komang Agus Adi Swara Putra, I. Made Dana, "Study of Optimal Portfolio Performance Comparison: Single Index Model and Markowitz Model on LQ45 Stocks in Indonesia Stock Exchange," vol.4, pp. 237-244, 2020.
- [2] T.Nandan and N.Srivastava, "Construction of Optimal Portfolio Using Sharpe's Single Index Model: An Empirical Study on Nifty 50 Stock".[Online].4(2).pp.743-83.Available:DOI:10.18231/2394-2770.2017.0010
- [3] D. Moreno, P. Marco and I. Olmeda, "Risk forecasting models and optimal portfolio selection." [Online].37(11).pp.1267-1281 Available: DOI: 10.1080/00036840500109142, .(February 2007).

- [4] J.Varghese and A.Joseph, "A Comparative Study on Markowitz Mean-Variance Model and Sharpe's Single Index Model in the Context of Portfolio Investment," vol.3, pp. 36-41, May 2018.
- [5] S.Ritika, T. Prabhat Kumar, " A Comparison of Markowitz and Sharpe's Model of Portfolio Analysis,"vol.6,pp.18-24, Jan-Jun 2017.
- [6] I. Nurhayati, E. endri, T. Suharti, R.S.Aminda, and L.Muniroh. "The impact of COVID-19 on formation and evaluation of portfolio performance: A case of Indonesia". [Online].18(3).pp.63-73.Available:DOI:10.21511/imfi. 18(3).2021.06
- [7] H.Rahyuda, "Analysis of LQ45 share portfolio on Quadrimester I during the Covid-19 pandemic." [Online].8(1).pp.9-18.Available: DOI:10.5267/j.ac.2021.6.011
- [8] M.R.Sarker, "Markowitz Portfolio Model: Evidence from Dhaka Stock Exchange in Bangladesh,"vol.8, pp.68-73, April 2013.
- [9] A. Gajera and K. Thakar, "Constructing an optimal portfolio using Sharpe's single index model," vol.4, pp.253-262, 2014.
- [10] C.A.Shah, "Construction of Optimal Portfolio Using Sharpe Index Model & Camp for BSE Top 15 Securities,"vol.2, pp.168-178, June 2015.